

关于 Mycielski 图补图的一些指标的结果

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$$S^*(G) = \sum_{\{v_i, v_j\} \in V(G)} d_G(v_i, v_j) (d_G(v_i) d_G(v_j))$$

Vukicevic

G

Lanzhou

¹⁰

$$Lz(G) = \sum_{u \in V(G)} \bar{d}_u d_u^2$$

$(n - 1)M_1(G) - F(G)$, n G u G \bar{G} u Lanzhou $Lz G$
 11-13 $M_1 G$ G Zagreb $F G$ G Forgotten

$$F(G) = \sum_{v_i \in V(G)} d_G^3(v_i) = \sum_{v_i, v_j \in E(G)} d_G^2(v_i) + d_G^2(v_j)$$

14 Mycielski Schultz Gutman
 Mycielski Schultz Gutman
 Mycielski Lanzhou

$G = (V(G), E(G))$ $V(G)$ $E(G)$ G $d_G(u)$ u
 $d_G(u, v)$ u v
 G \bar{G} $V(G)$ G \bar{G}
 G Mycielski ¹⁵ $\mu(G)$ V V $\{u\}$ $V = \{x : x \in V\}$ E $\{xy : xy \in E\}$
 $\{x u : x \in V\}$ x x u $\mu(G)$
 G n m

$$\sum_{\{v_i, v_j\} \in V(G)} (d_G(v_i) + d_G(v_j)) = 2m(n - 1)$$

G n m

$$\bar{M}_1(G) = \sum_{v_i, v_j \in E(G)} (d_G(v_i) + d_G(v_j)) = 2m(n - 1) - M_1(G)$$

G n m

$$\bar{M}_2(G) = 2m^2 - M_2(G) - \frac{1}{2} M_1(G)$$

G n m Mycielski $\mu(G)$ Mycielski $\bar{\mu}(G)$
 u $\{u, v\}$
 1 u $V(\mu(G))$ Mycielski $\mu(G)$ Mycielski $\bar{\mu}(G)$ u

$$d_{\mu(G)}(u) = \begin{cases} 2d_G(v_i), & u = v_i \\ d_G(v_i) + 1, & u = x_i \\ n, & u = z \end{cases}$$

$$d_{\bar{\mu}(G)}(u) = \begin{cases} 2n - 2d_G(v_i), & u = v_i \\ 2n - 1 - d_G(v_i), & u = x_i \\ n, & u = z \end{cases}$$

2 $\{u, v\}$ $V(\mu(G))$ Mycielski $\mu(G)$ Mycielski $\bar{\mu}(G)$ u v

$$(i) d_{\mu(G)}(u,v) = \begin{cases} 2, & u = x_i, v = x_j \\ d_G(v_i, v_j), & u = v_i, v = v_j, d_G(v_i, v_j) = 3 \\ 4, & u = v_i, v = v_j, d_G(v_i, v_j) = 4 \end{cases}$$

$$(ii) d_{\mu(G)}(u,v) = \begin{cases} 2, & u = v_i, v = x_j, i = j \\ d_G(v_i, v_j), & u = v_i, v = x_j, i \neq j, d_G(v_i, v_j) = 2 \\ 3, & u = v_i, v = x_j, i \neq j, d_G(v_i, v_j) = 3 \end{cases}$$

$$(iii) d_{\mu(G)}(u,v) = \begin{cases} 1, & u = x_i, v = z \\ 2, & u = v_i, v = z \end{cases}$$

$$(iv) d_{\mu(G)}(u,v) = \begin{cases} 1, & u = v_i, v = v_j, d_G(v_i, v_j) > 1, v_i v_j \in E(G) \\ 2, & u = v_i, v = v_j, d_G(v_i, v_j) = 1, v_i v_j \in E(G) \\ 1, & u = x_i, v = x_j \end{cases}$$

$$(v) d_{\mu(G)}(u,v) = \begin{cases} 1, & u = v_i, v = x_j, i = j \\ 1, & u = v_i, v = x_j, i \neq j, d_G(v_i, v_j) > 1 \\ 2, & u = v_i, v = x_j, i \neq j, d_G(v_i, v_j) = 1 \end{cases}$$

$$(vi) d_{\mu(G)}(u,v) = \begin{cases} 1, & u = v_i, v = z \\ 2, & u = x_i, v = z \end{cases}$$

$\mu(G)$	u	v	4	$\bar{\mu}(G)$	u	v	2
G	n	m		Mycielski	$\bar{\mu}(G)$	Schultz	
$S(\bar{\mu}(G)) = 8n^3 + 3n^2 - n - 4m - 5M_1(G).$							
$\mu(G)$	u	v		1	$\{v_i, v_j\}$	G	2
G	$3 \{v_i, x_j\}$	G	G	4	$\{v_i, z\}$	G	5
$\{x_i, z\}$	G						
1	u	v	1	Schultz			
				$d_{\bar{\mu}(G)}(v_i, v_j)(d_{\bar{\mu}(G)}(v_i) + d_{\bar{\mu}(G)}(v_j))$			
				$\sum_{\substack{\{v_i, v_j\} \\ v_i v_j \in E(G)}} d_{\bar{\mu}(G)}(v_i, v_j)(d_{\bar{\mu}(G)}(v_i) + d_{\bar{\mu}(G)}(v_j)) + \sum_{\substack{d_G(v_i, v_j) > 1 \\ v_i v_j \in E(G)}} d_{\bar{\mu}(G)}(v_i, v_j)(d_{\bar{\mu}(G)}(v_i) + d_{\bar{\mu}(G)}(v_j))$			
				$= \sum_{\substack{d_G(v_i, v_j) = 1 \\ v_i v_j \in E(G)}} 2[2n - 2d_G(v_i) + 2n - 2d_G(v_j)] + \sum_{\substack{d_G(v_i, v_j) > 1 \\ v_i v_j \in E(G)}} [2n - 2d_G(v_i) + 2n - 2d_G(v_j)]$			
				$= \sum_{v_i v_j \in E(G)} 2(4n - 2(d_G(v_i) + d_G(v_j))) + \sum_{v_i v_j \in E(G)} (4n - 2(d_G(v_i) + d_G(v_j)))$			
				$= 2n^3 - 2n^2 + 4m - 2M_1(G)$			
2	u	v	2	Schultz			
				$d_{\bar{\mu}(G)}(x_i, x_j)(d_{\bar{\mu}(G)}(x_i) + d_{\bar{\mu}(G)}(x_j))$			
				$\sum_{\substack{\{x_i, x_j\} \\ v(X)}} [2n - 1 - d_G(v_i)] + [2n - 1 - d_G(v_j)]$			

$$= \sum_{\{x, x_j\} \subseteq V(X)} 2(2n - 1) - (d_G(v_i) + d_G(v_j))$$

$$= 2n^3 - 3n^2 + (1 - 2m)n + 2m$$

3 u v 3 Schultz

i = j

$$\sum_{v_i \in V(G), x_j \in V(X)} d_{\bar{\mu}(G)}(v_i, x_j) (d_{\bar{\mu}(G)}(v_i) + d_{\bar{\mu}(G)}(x_j)) = \sum_{v_i \in V(G)} 1(2n - 2d_G(v_i) + 2n - 1 - d_G(v_i)) = 4n^2 - n - 6m$$

i j

$$\sum_{v_i \in V(G), x_j \in V(X)} d_{\bar{\mu}(G)}(v_i, x_j) (d_{\bar{\mu}(G)}(v_i) + d_{\bar{\mu}(G)}(x_j))$$

$$= \sum_{\substack{v_i \in V(G) \\ v_j \in V(G), x_j \in V(X)}} 2 \left[(4n - 1) - (d_G(v_i) + d_G(v_j)) - d_G(v_i) \right] \quad 1$$

$$+ \sum_{\substack{v_i \in V(G) \\ v_j \in V(G), x_j \in V(X)}} \left[(4n - 1) - (d_G(v_i) + d_G(v_j)) - d_G(v_i) \right] \quad 2$$

(1)

$$4m(4n - 1) - 2 \sum_{\substack{v_i \in V(G) \\ v_j \in V(G), x_j \in V(X)}} (d_G(v_i) + d_G(v_j)) - 2 \sum_{\substack{v_i \in V(G) \\ v_j \in V(G), x_j \in V(X)}} d_G(v_i) \quad \hat{a}$$

$$\begin{aligned}
&= \sum_{\substack{d(v_i, v_j) = 1 \\ v_i, v_j \in E(G)}} 2[(2n - 2d_G(v_i))(2n - 2d_G(v_j))] + \sum_{\substack{d(v_i, v_j) > 1 \\ v_i, v_j \in E(G)}} (2n - 2d_G(v_i))(2n - 2d_G(v_j)) \\
&= \sum_{v_i, v_j \in E(G)} 2[4n^2 - 4n(d_G(v_i) + d_G(v_j)) + 4d_G(v_i)d_G(v_j)] \\
&\quad + \sum_{v_i, v_j \in E(G)} [4n^2 - 4n(d_G(v_i) + d_G(v_j)) + 4d_G(v_i)d_G(v_j)] \\
&= 8m^2 - 8nM_1(G) + 8M_2(G) + 4n^2(C_n^2 - m) - 4n\overline{M}_1(G) + 4\overline{M}_2(G) \\
&= 2n^4 - 2n^3 - 4mn^2 + 8mn - (4n + 2)M_1(G) + 4M_2(G)
\end{aligned}$$

2 u v 2

Schultz

$$\begin{aligned}
d_{\bar{\mu}(G)}(x_i, x_j) \left(d_{\bar{\mu}(G)}(x_i) d_{\bar{\mu}(G)}(x_j) \right) &= \sum_{\{x_i, x_j\} \in V(X)} [2n - 1 - d_G(v_i)][2n - 1 - d_G(v_j)] \\
&= \sum_{\{x_i, x_j\} \in V(X)} (2n - 1)^2 - (2n - 1)(d_G(v_i) + d_G(v_j)) + d_G(v_i)d_G(v_j) \\
&= 2n^4 - 4n^3 + \left(\frac{5}{2} - 4m\right)n^2 + \left(6m - \frac{1}{2}\right)n + 2m^2 - 2m - \frac{1}{2}M_1(G)
\end{aligned}$$

3 u v 3

Schultz

i = j

$$\begin{aligned}
d_{\bar{\mu}(G)}(v_i, x_j) \left(d_{\bar{\mu}(G)}(v_i) d_{\bar{\mu}(G)}(x_j) \right) &= \sum_{v_i, v_j \in V(G)} (2n - 2d_G(v_i))(2n - 1 - d_G(v_i)) \\
&= \sum_{v_i, v_j \in V(G)} (4n^2 - 2n) - (6n - 2)d_G(v_i) + 2d_G^2(v_i) \\
&= 4n^3 - 2n^2 - 12mn + 4m + 2M_1(G)
\end{aligned}$$

i j

$$\begin{aligned}
& d_{\bar{\mu}(G)}(v_i, x_j) \left(d_{\bar{\mu}(G)}(v_i) d_{\bar{\mu}(G)}(x_j) \right) \\
&= \sum_{\substack{v_i, v_j \in E(G) \\ v_i, v_j \in V(G), x_j \in V(X)}} 2[(2n - 2d_G(v_i))(2n - 1 - d_G(v_i))] \quad 3 \\
&\quad + \sum_{\substack{v_i, v_j \in E(G) \\ v_i, v_j \in V(G), x_j \in V(X)}} (2n - 2d_G(v_i))(2n - 1 - d_G(v_i)) \quad 4
\end{aligned}$$

3

$$\begin{aligned}
& 2m(8n^2 - 4n) - 4n \sum_{\substack{v_i, v_j \in E(G) \\ v_i, v_j \in V(G), x_j \in V(X)}} d_G(v_i) + d_G(v_j) + (2 - 4n) \sum_{\substack{v_i, v_j \in E(G) \\ v_i, v_j \in V(G), x_j \in V(X)}} d_G(v_i) + 4 \sum_{\substack{v_i, v_j \in E(G) \\ v_i, v_j \in V(G), x_j \in V(X)}} d_G(v_i)d_G(v_j) \\
&= 16m^2 - 8mn + (2 - 12n)M_1(G) + 8M_2(G)
\end{aligned}$$

4

$$\begin{aligned}
& 2(4n^2 - 2n)(C_n^2 - m) - 2n \sum_{\substack{v_i, v_j \in E(G) \\ v_i, v_j \in V(G), x_j \in V(X)}} d_G(v_i) + d_G(v_j) + (2 - 2n) \sum_{\substack{v_i, v_j \in E(G) \\ v_i, v_j \in V(G), x_j \in V(X)}} d_G(v_i) + 2 \sum_{\substack{v_i, v_j \in E(G) \\ v_i, v_j \in V(G), x_j \in V(X)}} d_G(v_i)d_G(v_j) \\
&= 4n^4 - 6n^3 + (2 - 20m)n^2 + 20mn + 8m^2 - 4m + (6n - 4)M_1(G) - 4M_2(G)
\end{aligned}$$

4 u v 4

Schultz

$$d_{\bar{\mu}(G)}(v_i, Z) \left(d_{\bar{\mu}(G)}(v_i) d_{\bar{\mu}(G)}(Z) \right) = \sum_{v_i, v_j \in V(G), z \in Z} 1(2n - 2d_G(v_i))n = 4n^2 - 4mn$$

5 u v 5

Schultz

$$d_{\bar{\mu}(G)}(x_i, Z) \left(d_{\bar{\mu}(G)}(x_i) d_{\bar{\mu}(G)}(Z) \right) = \sum_{x_i, v_j \in V(G), z \in Z} 2(2n - 1 - 2d_G(v_i))n = 4n^3 - 2n^2 - 4mn$$

Mycielski

 $\bar{\mu}(G)$

Schultz

$$S(\bar{\mu}(G)) = 8n^4 - 4n^3 + \left(\frac{9}{2} - 12m\right)n^2 + \left(6m - \frac{1}{2}\right)n + 10m^2 - 2m - \left(10n + \frac{5}{2}\right)M_1(G) + 8M_2(G)$$

Mycielski

Schultz

Schultz

i $S(\bar{\mu}(K_n)) = 3n^3 + 11n^2 - 4n, (n \geq 2);$

ii $S(\bar{\mu}(K_{st})) = 8s^3 + (19t + 3)s^2 + (19t^2 + 2t + 1)$

- 2012 31 51-55.
- 5 HUA H B ASHRAFI A R ZHANG L B. More on Zagreb Coindices of Graphs J . Faculty of Sci and Math 2012 6 1210- 1220.
- 6 ASHRAFIA R DOŠLI T HAMZEH A. The Zagreb Coindices of Graph Operations J . Discrete Appl. Math 2010 158 1571- 1578.
- 7 MATEJI M MILOVANOVI E MILOVANOVI I et al. A Note on the First Zagreb Index and Coindex of Graphs J . Commun. Comb. Opti 2021 06 41- 51.
- 8 MILOVANOVI E MILOVANOVI I. Sharp Bounds for the First Zagreb Index and First Zagreb Coindex J . Miskolc Math Notes 2015 16 02 1017- 1024.
- 9 GUTMAN I. Degree- based Topological Indices J . Croat Chem Acta 2013 86 04 351- 361.
- 10 VUKICEVIC D LI Q SEDLAR J et al. Lanzhou Index J . MATCH Commun. Math. Comput. Chem 2018 80 03 863- 876.
- 11 DEHGARDI N LIU J. Lanzhou Index of Trees with Fixed Maximum Degree J . MATCH Commun. Math. Comput. Chem 2021 86 3- 10.
- 12 FURTULA B GUTMAN I. A Forgotten Topological Index J . J Math Chem 2015 53 04 1184- 1190.
- 13 LIU J B MATEJI M MILOVANOVI E et al. Some New Inequalities for the Forgotten Topological Index and Coindex of Graphs J . MATCH Commun. Math. Comput. Chem 2020 84 719- 738.
- 14 . Mycielski J . 2020 46 04 63- 68.
- 15 RUDNICKI P STEWART L. The Mycielskian of a Graph J . Formalized Math 2011 19 27- 34.

Results on Some Indices of Complements of Mycielski Graphs

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Topology index is a mathematical descriptor of molecular structure, which digitizes the structural characteristics of molecules such as shape, size, and branching. It is easy to calculate, has objective values, and is not easily limited by experience and experiments. The study of topological index graph invariants is currently one of the most active research areas in chemical graph theory, which can be used to describe and predict the physicochemical or pharmacological properties of organic compounds. This article studies two types of degree distance metrics for the complement of Mycielski graphs: Schultz index and modified Schultz index. At the same time, expressions for the Lanzhou index of Mycielski graphs and their complement graphs of some special graphs are also provided.